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Nonlinear vibration experiment: Clamped circular elastic plate with granular material loading

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Abstract:

Experiments using a soil-plate-oscillator (SPO) involve a vertical cylindrical column of granular medium (masonry sand, glass spheres, uncooked brown rice, un-popped popcorn kernels, or even "Toasty Oats" ™ cereal) that is supported by an air-backed thin circular elastic acrylic plate (20.3 cm diam and 3.2 mm thick) that is rigidly clamped to the bottom of a thick-walled aluminum tube. The soil column is driven from below using an electrodynamic system. Here an AC coil placed on axis and below the plate, drives a 1 cm diam 1.5 cm long rare earth magnet that is fastened to the underside center of the plate. The coil is electrically driven by an amplified swept sinusoidal slowly varying chirp. A small accelerometer attached to the magnet is used to measure the vibration. In nonlinear tuning curve experiments the resonant frequency decreases significantly with increased amplitude – representing a softening in the nonlinear system. For fixed amplitude the resonant frequency vs. the granular medium mass loading (over the plate) reaches a minimum and then increases with increased loading due to the granular medium's flexural stiffness – which overcomes the mass loading effects. For water loading, the frequency always decreases since there is no bending stiffness.

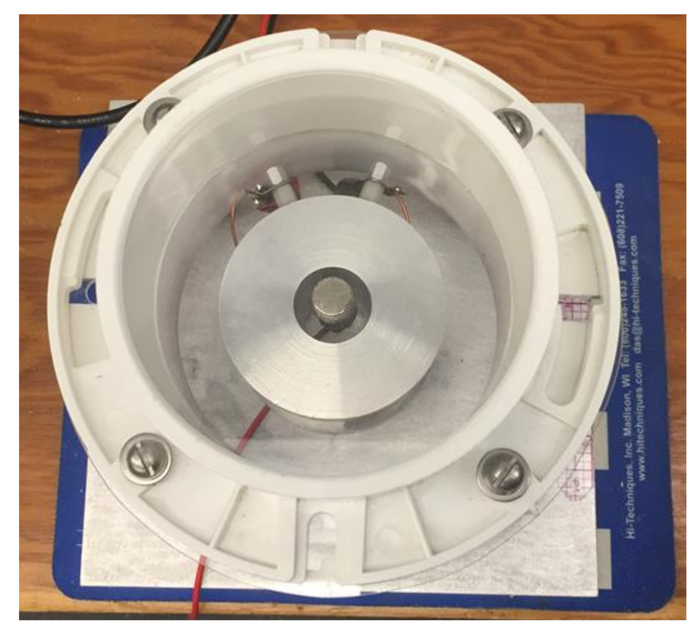
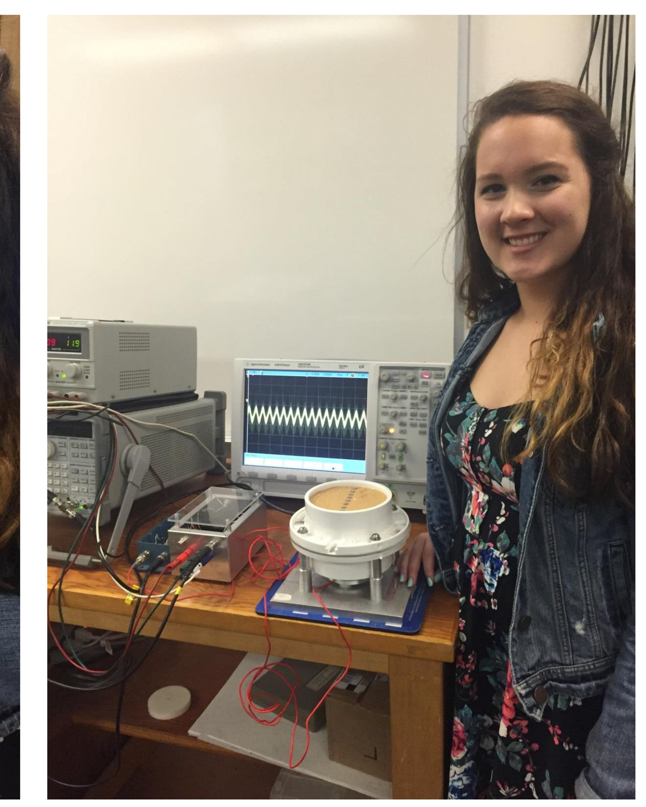
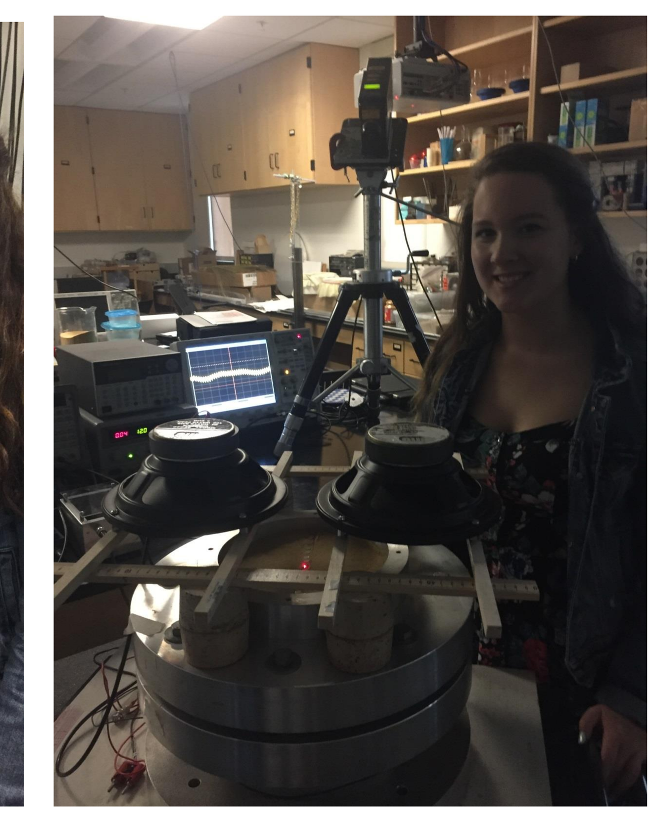


Fig. (A,B,C) 4.5 inch diam SPO (two PVC flanges, 1/8 inch clamped acrylic plate, rare earth magnet (1cm diam) and accelerometer on underside. Fig. (D) 8.0 inch diam alum. SPO.





Photos: Soil plate oscillator experiments performed with dry sifted masonry sand.

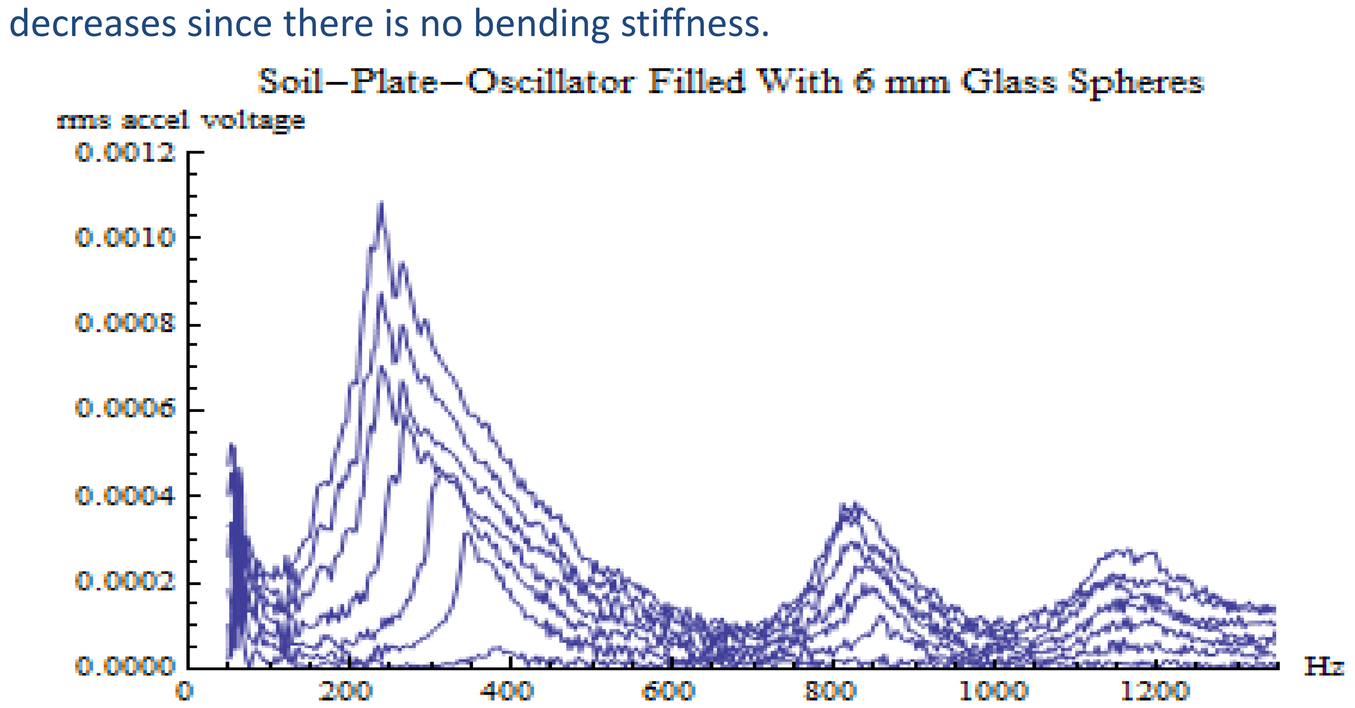
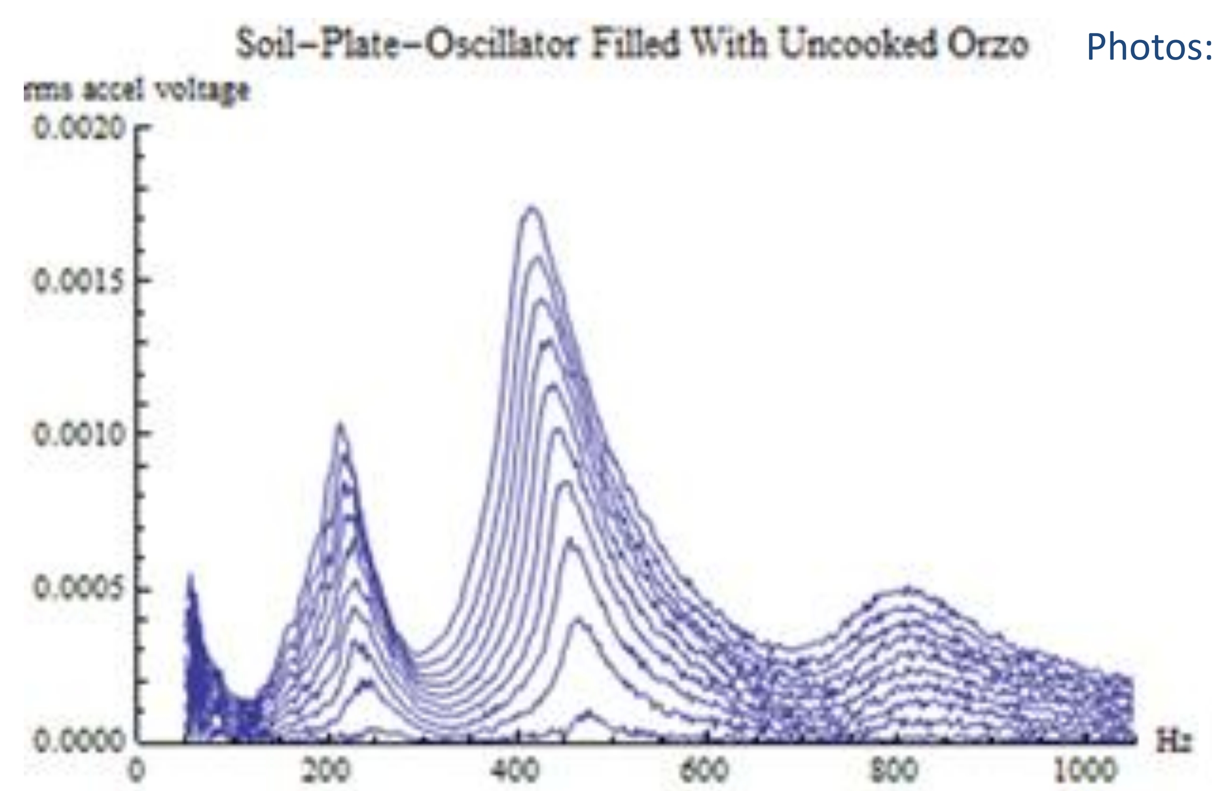
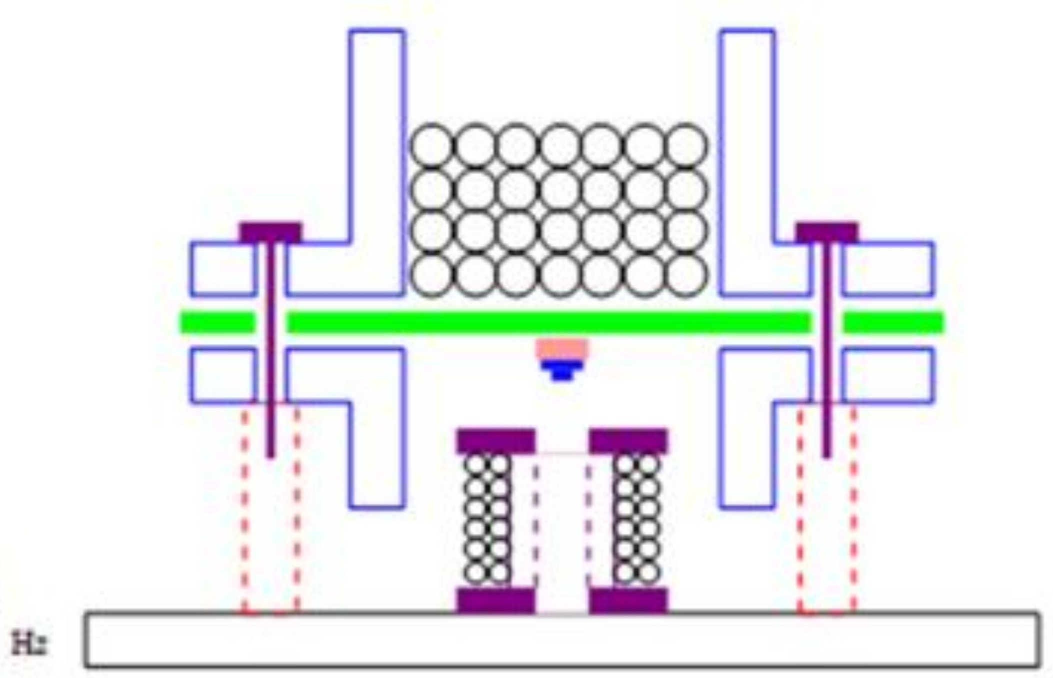


Figure 1. A set of tuning curve responses of a Soil Plate Oscillator (SPO) filled with 6 mm glass spheres. The clamped acrylic 1/8 inch plate is 4.5 inch in diam.





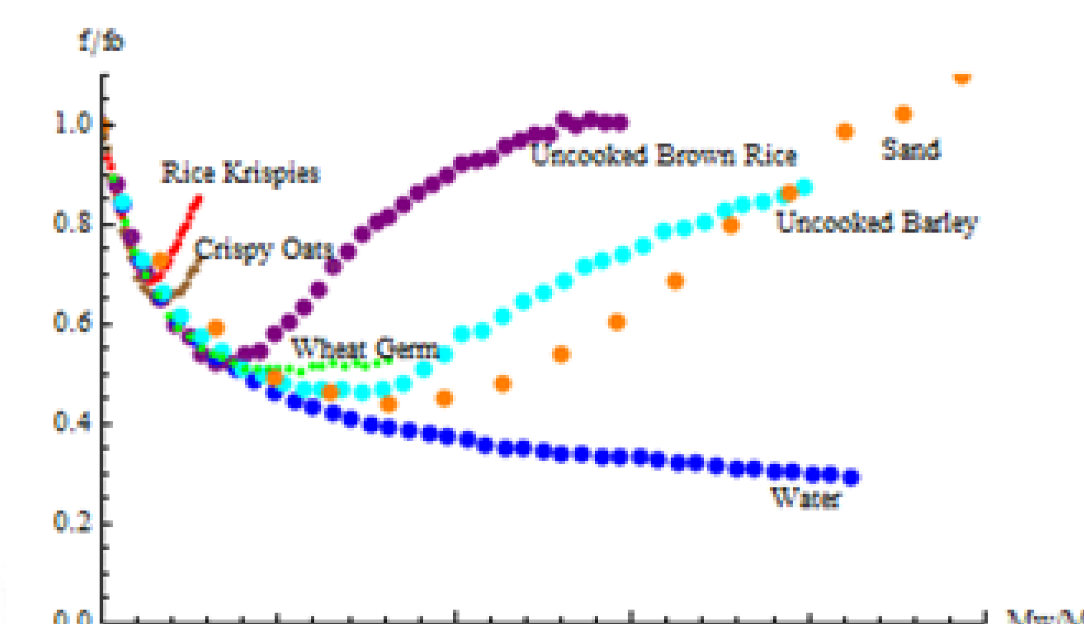
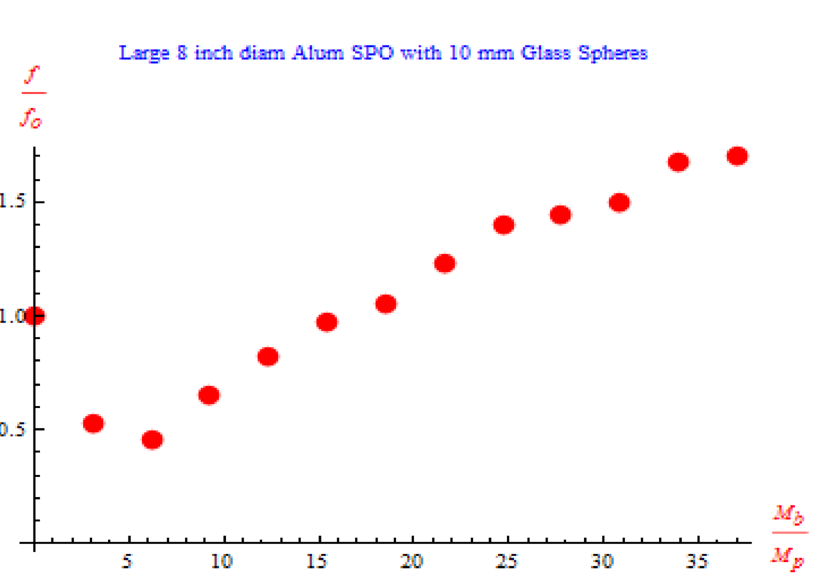


Figure 2. (A) A set of tuning curve responses of a Soil Plate Oscillator (SPO) filled with uncooked Orzo™ shows three distinct peaks of the accelerometer rms voltage signal. Here, an amplified swept sinusoidal tone from 50 Hz to 1050 Hz drives the AC coil shown in Fig. B which generates a localized force on the magnet centered underneath the plate. For increased drive amplitude the resonant frequencies exhibit a "softening" due to the nonlinearity of the granular medium. (B) A diagram of a Soil Plate Oscillator filled with granular medium from a side angle, cut down the middle to show a cross sectional view of the apparatus. (C) A graph showing the effects of frequency versus mass loading of the clamped elastic plate using Rice Krispies™, Crispy Oats™, wheat germ, uncooked brown rice, mansonry sand, uncooked barley, and water. The y-axis is the normalized frequency f/fo and the x-axis is the granular mass normalized to the clamped acrylic 1/8 inch by 4.5 inch diam plate mass of 37.0 grams. The SPO with increased water loading always exhibits a decrease in its resonant frequency. However, with dry granular media, the flexural stiffness of the loaded circular plate will eventually dominate the mass loading effects and the resonant frequency will then show an increase in frequency with further loading. The accelerometer calibration is 1 mV corresponds to 1.14 m/s². The SPO column is 2 inches high . Fig. 2 (B) is not draw to scale.



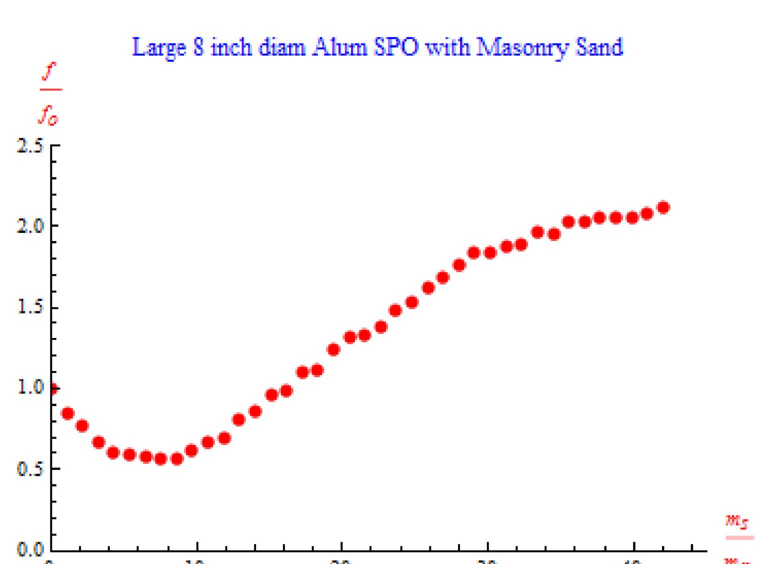
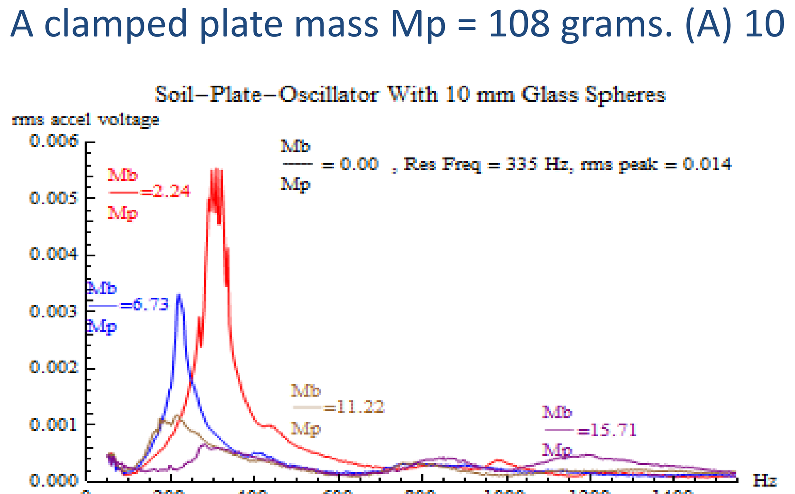


Figure 3. Comparing resonant frequency vs. mass loading in an 8 inch diam SPO with A clamped plate mass Mp = 108 grams. (A) 10 mm glass spheres, (B) sifted mansonry sand



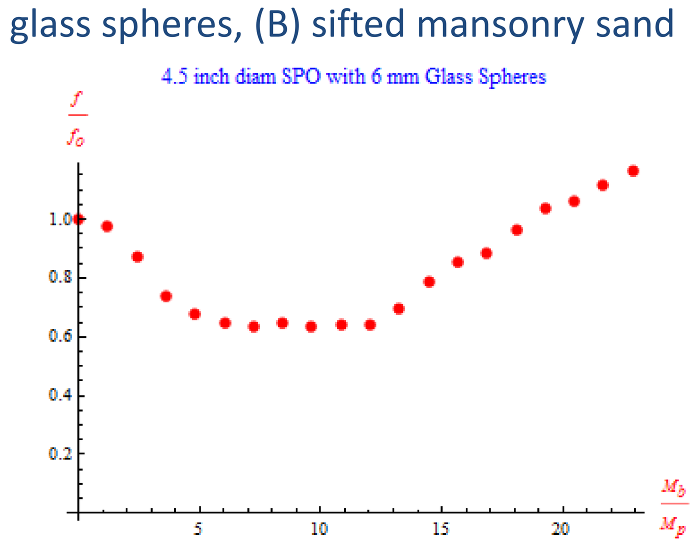


Figure 4. Comparing resonant frequency vs. mass loading effects in a 4.5 inch diam SPO

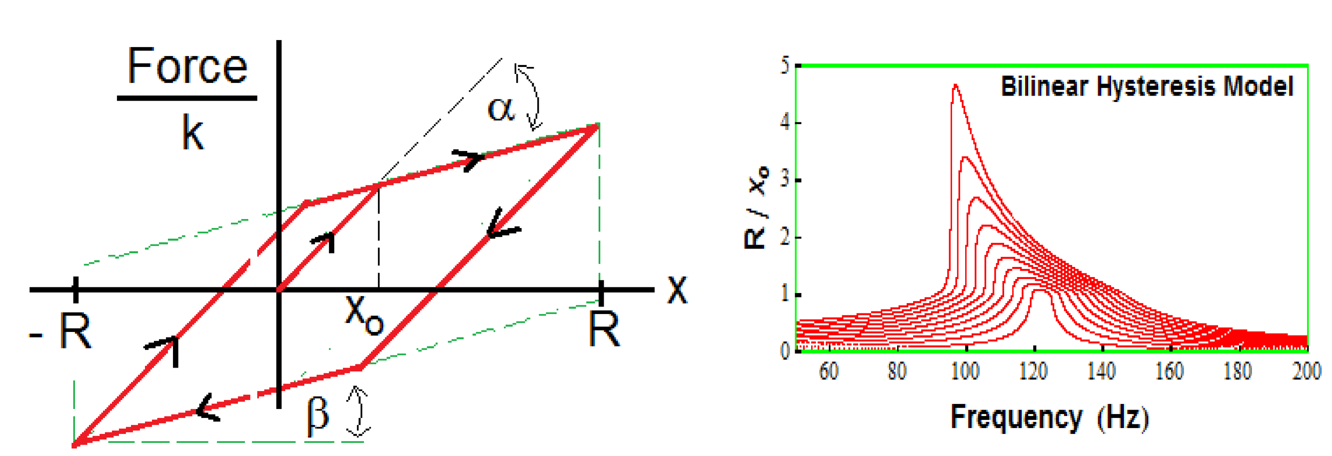


Figure 5. A theoretical model of sinusoidal excitation of a mass-spring oscillator exhibiting bilinear hysteresis (diagram on left) is used to predict the tuning curve response (on the right). See Ref 1. Here $\tan(\beta) = (1-\mu)$ and $\tan(\alpha) = (\mu/(2-\mu))$. Parameters f = {0.1, 0.2, 0.3,…,1.0} with $\mu = 0.44$ seem to fit the results for a buried acrylic drum-like mine simulant described in Ref 3. Caughy (1960) solved the forced vibration of a mass m connected to a spring which has this form of bilinear hysteresis. The equation of motion is $m\, d^2x/dt^2 + k\, F(x,m,t) = F_o \cos(\omega t)$ where $F(x,\mu,t)$ is given in Fig. 5. If $k/m = \omega_o^2$, $\tau = \omega_o t$, $F_o / k = x_s$, and $\eta = \omega / \omega_o$ then the solution in the form $x(t) = R \cos(\eta \tau + \phi)$ is solved for R and f, using the method of slowly varying parameters. Using $f = x_s/x_o$, $A = R/x_o$, and $\theta^* = \cos^{-1}(1-2/A)$, the solution is $\eta = (1 + \mu X)^{1/2}$ where $X = -\{1-\theta^*/\pi + (\sin 2\theta^*)/2\pi \ +/- \ [(f/A)^2 - (\sin^2 \theta^*/\pi)^2]^{1/2}\}$ for A >1 and X= +/- (f/A) for A < 1 .

References & Related Material:
1. T. K. Caughey, Transactions of the ASME, J. Applied Mech., 27, (1960), 640-643.
2. L. A. Ostrovsky and P. A. Johnson, "Dynamic Nonlinearity Elasticity and geomaterials,"Revista Del Nouvo Cinmento, 24, serie 4, No. 7, 1-46 (2001).
3. M. S. Korman, D. V. Duong and A. E. Kalsbeck, A.I.P. Conference Proceedings, 20th ISNA, Ecully, France, 29 June-3 July 2015, "Electrodynamic Soil Plate Oscillator: Modeling Nonlinear Mesoscopic Elastic Behavior and Hysteresis in Nonlinear Acoustic Landmine Detection," (080003-1 to 8).